STEADY FREE CONVECTION IN VISCOPLASTIC LIQUIDS

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The formulation and solution are given for the external problem of the steady free convection of a viscoplastic liquid.

The widespread application of non-Newtonian liquids in various processes of chemical technology and power engineering is responsible for the heightened interest in processes of heat exchange in these media, particularly during free-convective motion. Up to now the problem of free convection under the conditions of the external problem has mainly been examined only in nonplastic liquids [1, 2]. However, many industrially important media are characterized by a finite yield point, i.e., they display plastic properties. In the single report known to us [3] which is devoted to an examination of free convection in a viscoplastic liquid under the conditions of the external problem the formulation of the problem is incorrect, since it does not take into account a specific property of these media: the finiteness of the region of flow. The formulation and the method of solution of problems of free-convective motion in viscoplastic liquids are given in the present report.

Let us examine the free convection near a vertical cylinder of constant radius submerged in a Schvedo–Bingham viscoplastic liquid. It follows from physical considerations that with steady free-convective motion of a viscoplastic liquid the entire region under consideration in the general case can be divided into five zones: I) viscoplastic flow ($\partial u/\partial y > 0$); II) quasisolid motion ($\partial u/\partial y = 0$); III) viscoplastic flow ($\partial u/\partial y < 0$); IV) viscoplastic flow ($\partial u/\partial y \leq 0$, $\Theta = 0$); V) stationary liquid ($u = 0$) (Fig. 1).

The dimensionless equations of motion, continuity, and energy in the boundary-layer approximation for a coordinate system connected with the surface of the body have the form (it is assumed that the thickness of the boundary layer is much less than the radius of the cylinder):

Zone I. ($0 \leq y < y_1$):

$$u_1 \frac{\partial u_1}{\partial x} + v_1 \frac{\partial u_1}{\partial y} = \frac{\partial^2 u_1}{\partial y^2} + R + \Theta_1;$$

$$\frac{\partial u_1}{\partial x} + \frac{\partial v_1}{\partial y} = 0;$$

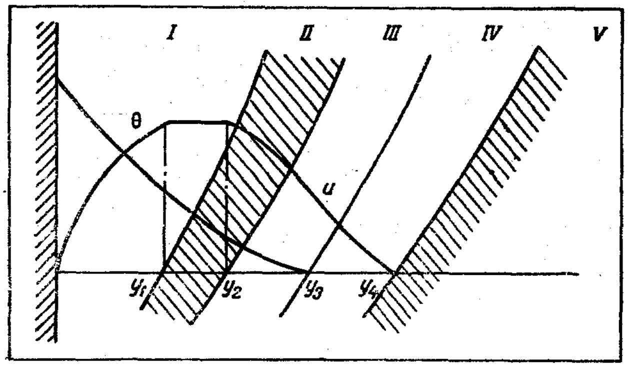

Fig. 1. Diagram of flow.

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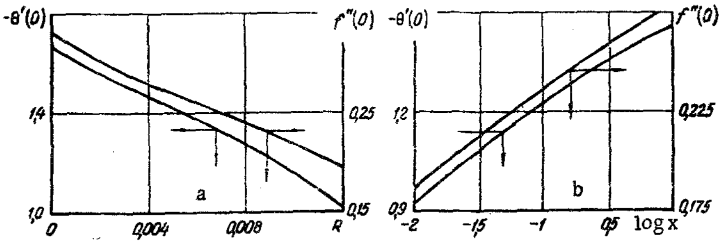

Fig. 2. Characteristic curves of heat exchange and friction as functions of: a) the plasticity parameter R [$A = (1.33 \cdot 10^5)^{1/4}$]; b) the longitudinal coordinate ($R = 0.004$, $Gr = 10^5$).

$$u_1 \frac{\partial \Theta_1}{\partial x} + v_1 \frac{\partial \Theta_1}{\partial y} = \frac{1}{\Pr} \cdot \frac{\partial^2 \Theta_1}{\partial y^2}, \tag{1}$$

boundary conditions:

$$u_1(x,\ 0) = 0; \quad v_1(x,\ 0) = 0; \quad \Theta_1(x,\ 0) = 1; \quad \frac{\partial u_1(x,\ y_1)}{\partial y} = 0; \tag{2}$$

Zone II. ($y_1 \le y \le y_2$):

$$\begin{gathered} u_2 \frac{du_2}{dx} = \mathrm{Gr}^{1/4} \frac{\partial \tau}{\partial y} + \tau + \Theta_2; \\ \frac{du_2}{dx} + \frac{\partial v_2}{\partial y} = 0; \\ u_2 \frac{\partial \Theta_2}{\partial x} + v_2 \frac{\partial \Theta_2}{\partial y} = \frac{1}{\Pr} \cdot \frac{\partial^2 \Theta_2}{\partial y^2}, \end{gathered} \tag{3}$$

boundary conditions:

$$\tau(x,\ y_1) = R; \quad \Theta_2(x,\ y_1) = \Theta_1(x,\ y_1); \quad \frac{\partial \Theta_2(x,\ y_1)}{\partial y} = \frac{\partial \Theta_1(x,\ y_1)}{\partial y}; \quad v_2(x,\ y_1) = v_1(x,\ y_1); \quad \tau(x,\ y_2) = -R; \tag{4}$$

Zone III. ($y_2 < y \le y_3$):

$$\begin{gathered} u_3 \frac{\partial u_3}{\partial x} + v_3 \frac{\partial u_3}{\partial y} = \frac{\partial^2 u_3}{\partial y^2} - R + \Theta_3; \\ \frac{\partial u_3}{\partial x} + \frac{\partial v_3}{\partial y} = 0; \\ u_3 \frac{\partial \Theta_3}{\partial x} + v_3 \frac{\partial \Theta_3}{\partial y} = \frac{1}{\Pr} \cdot \frac{\partial^2 \Theta_3}{\partial y^2}, \end{gathered} \tag{5}$$

boundary conditions:

$$\begin{gathered} u_3(x,\ y_2) = u_1(x,\ y_1); \quad v_3(x,\ y_2) = v_2(x,\ y_2); \quad \Theta_3(x,\ y_2) = \Theta_2(x,\ y_2); \\ \frac{\partial u_3(x,\ y_2)}{\partial y} = 0; \quad \frac{\partial \Theta_3(x,\ y_2)}{\partial y} = \frac{\partial \Theta_2(x,\ y_2)}{\partial y}; \\ \Theta_3(x,\ y_3) = 0; \quad \frac{\partial \Theta_3(x,\ y_3)}{\partial y} = 0; \end{gathered} \tag{6}$$

Zone IV. ($y_3 < y \le y_4$):

$$u_4 \frac{\partial u_4}{\partial x} + v_4 \frac{\partial u_4}{\partial y} = \frac{\partial^2 u_4}{\partial y^2} - R; \quad \frac{\partial u_4}{\partial x} + \frac{\partial v_4}{\partial y} = 0, \tag{7}$$

boundary conditions:

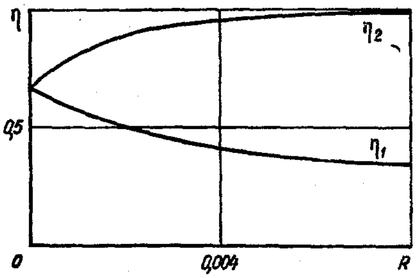

Fig. 3. Coordinates η_1 and η_2 of boundaries as functions of the plasticity parameter $R [A = (1.33 \cdot 10^5)^{1/4}]$.

$$u_4(x, y_3) = u_3(x, y_3); \quad \frac{\partial u_4(x, y_3)}{\partial y} = \frac{\partial u_3(x, y_3)}{\partial y};$$
$$v_4(x, y_3) = v_3(x, y_3); \quad u_4(x, y_4) = 0; \quad \frac{\partial u_4(x, y_4)}{\partial y} = 0. \tag{8}$$

The results of [4, 5] on the forced flow of viscoplastic liquids were used in the derivation of the system of equations (1)-(8).

The parameter $R = \tau_0/\rho r_0 \beta g(T_0 - T_\infty)$ which was introduced characterizes the ratio of the plastic (yield point) and buoyant (Archimedes) stresses. This parameter has a determining significance in the free-convective motion of viscoplastic liquids.

Free convection in viscoplastic liquids is characterized by the essentially finite region of the flow which, as is known, prevents one from finding completely self-similar solutions. A locally self-similar solution of the problem is constructed in the present work. This method is widely used in problems of the free-convective motion of purely viscous media [6, 7].

Through the introduction of new dependent and independent variables

$$\eta = \frac{y}{\left(\frac{4x}{3}\right)^{1/4}}; \quad \Psi_i = f_i(\eta, x)\left(\frac{4x}{3}\right)^{3/4} \quad (i = 1, 2, 3, 4),$$

where the stream functions are determined by the equations

$$u_i = \frac{\partial \Psi_i}{\partial y}; \quad v_i = -\frac{\partial \Psi_i}{\partial x} \quad (i = 1, 2, 3, 4),$$

the problem (1)-(8) in the approximation of local self-similarity is reduced to the following problem

Zone I. $(0 \le \eta < \eta_1)$:

$$f_1''' + f_1 f_1'' - \frac{2}{3}(f_1')^2 + R + \Theta_1 = 0;$$
$$\Theta_1'' + \Pr f_1 \Theta_1' = 0, \tag{9}$$

boundary conditions:

$$f_1 = 0; \quad f_1' = 0; \quad \Theta_1 = 1 \quad \text{at} \quad \eta = 0; \quad f_1'' = 0 \quad \text{at} \quad \eta = \eta_1; \tag{10}$$

Zone II. $(\eta_1 \le \eta \le \eta_2)$:

$$\tau' - \left\{\frac{2}{3}(f_2')^2 - \tau - \Theta_2\right\} A = 0;$$
$$\Theta_2'' + \Pr f_2 \Theta_2' = 0, \tag{11}$$

boundary conditions:

$$\tau = R; \quad \Theta_2 = \Theta_1; \quad \Theta_2' = \Theta_1' \quad \text{at} \quad \eta = \eta_1;$$
$$\tau = -R \quad \text{at} \quad \eta = \eta_2; \tag{12}$$

Zone III. $(\eta_2 < \eta \le \eta_3)$:

$$f_3''' + f_3 f_3'' - \frac{2}{3}(f_3')^2 - R + \Theta_3 = 0;$$
$$\Theta_3'' + \Pr f_3 \Theta_3' = 0, \tag{13}$$

boundary conditions:

$$f_3 = f_1(\eta_1) + f_1'(\eta_1)(\eta_2 - \eta_1); \quad f_3' = f_1'(\eta_1);$$
$$f_3'' = 0; \quad \Theta_3 = \Theta_2; \quad \Theta_3' = \Theta_2' \quad \text{at} \quad \eta = \eta_2;$$
$$\Theta_3 = 0; \quad \Theta_3' = 0 \quad \text{at} \quad \eta = \eta_3; \tag{14}$$

Zone IV. ($\eta_3 < \eta \le \eta_4$):

$$f_4''' + f_4 f_4'' - \frac{2}{3}(f_4')^2 - R = 0, \tag{15}$$

boundary conditions:

$$f_4 = f_3;\quad f_4' = f_3';\quad f_4'' = f_3'' \quad \text{at} \quad \eta = \eta_3;$$
$$f_4' = 0;\quad f_4'' = 0 \quad \text{at} \quad \eta = \eta_4, \tag{16}$$

where the prime denotes differentiation with respect to η, and $A = (4x/3Gr)^{1/4}$.

It follows from physical considerations that in the quasisolid zone $u_2 = \text{const}(y)$, from which $f_2'(\eta) = f_1'(\eta_1) = \text{const}$. Consequently,

$$f_2(\eta) = f_1(\eta_1) + f_1'(\eta_1)(\eta - \eta_1). \tag{17}$$

The numerical solution of problem (9)-(16) with allowance for the condition (17) was obtained on a Minsk-22 computer. Some results of the calculations are presented in Figs. 2-3. Both the heat exchange and the friction at the surface of the heater increase with greater distance from the leading edge of the cylinder (Fig. 2). In addition, it follows from Fig. 2b that the approximation of local self-similarity is well justified, since with a variation of two orders of magnitude in the longitudinal coordinate we have

$$\frac{\Delta[-\Theta'(x, 0)]x_1}{[-\Theta'(x, 0)]x_2} < 0.003;\quad \frac{\Delta f''(x, 0)x_1}{f''(x, 0)x_2} < 0.015.$$

Intensification of the plastic properties (an increase in the parameter R) leads to suppression of the free-convective motion, i.e., to a sharp decrease in both the heat-exchange and the friction characteristics (Fig. 2a), with the width of the quasisolid zone ($\Delta_1 = \eta_2 - \eta_1$) increasing as this happens (the rate of growth is especially noticeable for small R) (Fig. 3). It follows from physical considerations that the thickness of zone II remains finite upon approach to the leading edge, i.e., the motion takes place in a zone of nonzero thickness as $x \to 0$, which is an important difference from the flow pattern in a purely viscous medium. We should note that the presence of two quasisolid zones in the free-convective flow of viscoplastic media qualitatively differentiates this type of flow from external problems of forced convection. Moreover, in forced convection because of the infinite stresses in the region of the leading edge of the body the thickness of the zone in which the velocity differs from the velocity of the external flow (quiescence for the case of the movement of a body in a stationary medium) approaches zero, which once again differs qualitatively from the case of free-convective motion.

NOTATION

$x = x'/L$, $y = y'/L(Gr^{1/4})$, dimensionless coordinates; x', y', dimensional coordinates; $u = u'[L\beta g(T_0 - T_\infty)]^{-1/2}$; $v = v'[L\beta g(T_0 - T_\infty)]^{-1/2} Gr^{1/4}$, dimensionless velocities; L, characteristic size; β, volumetric expansion coefficient; $Pr = \mu_p c_p/\lambda$, modified Prandtl number; $Gr = (\rho/\mu_p)^2 L^3 \beta g(T_0 - T_\infty)$, Grashof number; $\Theta = (T - T_\infty)/(T_0 - T_\infty)$, dimensionless temperature; T_0, T_∞, temperature at wall and as $y \to \infty$, respectively; τ_0, yield point; μ_p, plastic viscosity.

LITERATURE CITED

1. A. J. Ede, An Introduction to Heat Transfer; Principles and Calculations, Pergamon Press, New York (1967).
2. J. D. Dale and A. F. Emery, Teploperedacha, No. 1 (1972).
3. J. Kleppe and W. J. Marner, Teploperedacha, No. 4 (1972).
4. I. M. Astrakhan, Izv. Akad. Nauk SSSR, Otd. Tekh. Nauk, Mekh. Mashinostr., No. 2 (1960).
5. Z. P. Shul'man and V. I. Baikov, BelSSR, Ser. Fiz.-Énerget. Nauk, No. 1 (1971).
6. V. Kubari and D. C. Pei, Int. J. Heat Mass Transfer, 11, 855 (1969).
7. G. Wilks, Int. J. Heat Mass Transfer, 16, No. 10 (1973).